

## Eigenvalues 3

Given an automorphism in  $\mathbb{P}_2$  the vector space of 2nd degree polynoms named  $f$ .

1. Obtain its kernel and image.

2. Knowing that  $x+1$  transforms into itself, that 3 transforms into -3 and that the determinant of the matrix is 6, obtain the eigenvalues of the automorphism.

3. knowing that  $L\{(x^2+x+1)\}$  is an eigenspace, obtain  $F_B$ .

Note: You cannot use information of later points of the problem in previous points.

$$1. \quad \text{Ker}(f) = \{0\} \quad \dim(\text{Ker}(f)) = 0$$

$$\text{Im}(f) = \mathbb{P}_2 \quad \dim(\text{Im}(f)) = 3$$

$$2. \quad S(-1) = \mathcal{L}\{1\} \quad M_0(-1) = 1$$

$$S(1) = \mathcal{L}\{x+1\} \quad M_0(1) = 1$$

$$S(-6) = \mathcal{L}\{x^2+x+1\} \quad M_0(-6) = 1$$

→ from 3.

$$3. \quad F_B ?$$

$$F = \begin{pmatrix} -6 & 0 & 0 \\ -7 & 1 & 0 \end{pmatrix}$$

$$f(1) = -1$$

$$f(x+1) = x+1 \rightarrow f(x) + \underbrace{f(1)} = x+1 \rightarrow f(x) = x+2$$

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$$f(x^2) = -6x^2 - 7x + 1$$

Eigenvalues 4 Given an endomorphism in  $\mathbb{R}^3$  where  $f(\bar{x}) = (x^1+x^2, x^1+x^3, x^2+x^3)$ .

1. Prove it's an Endomorphism.
2. Obtain  $\text{Ker}(f)$  and  $\text{Im}(f)$ .
3. Obtain  $S(\lambda_i)$   $\forall i$
4. Is  $f$  diagonalizable?

$$f: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$$

$$\forall \alpha, \beta \in \mathbb{R} \wedge \forall \bar{x}, \bar{y} \in \mathbb{R}^3$$

$$f \text{ is a homomorphism } \stackrel{?}{\iff} f(\alpha \bar{x} + \beta \bar{y}) \stackrel{?}{=} \alpha f(\bar{x}) + \beta f(\bar{y})$$

$$f(x^1, x^2, x^3) = (x^1+x^2, x^1+x^3, x^2+x^3)$$

$$f(\alpha \bar{x} + \beta \bar{y}) = f(\alpha(x^1, x^2, x^3) + \beta(y^1, y^2, y^3)) = f(\alpha x^1 + \beta y^1, \alpha x^2 + \beta y^2, \alpha x^3 + \beta y^3) =$$

$$\bar{x} = (x^1, x^2, x^3)_B$$

$$\bar{y} = (y^1, y^2, y^3)_B$$

$$B = \{\bar{e}_1, \bar{e}_2, \bar{e}_3\} \subset \mathbb{R}^3$$

$$= (\alpha x^1 + \beta y^1 + \alpha x^2 + \beta y^2, \alpha x^1 + \beta y^1 + \alpha x^3 + \beta y^3, \alpha x^2 + \beta y^2 + \alpha x^3 + \beta y^3) =$$

$$= (\alpha(x^1+x^2) + \beta(y^1+y^2), \alpha(x^1+x^3) + \beta(y^1+y^3), \alpha(x^2+x^3) + \beta(y^2+y^3)) =$$

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2.  $F_B = \begin{pmatrix} | & | & | \\ 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$   
 $f(\bar{e}_1) \quad f(\bar{e}_2) \quad f(\bar{e}_3)$   
 in  $B$

$\dim(\text{Im}(f)) = \text{Rg}(F_B) = 3 \rightarrow \dim(\text{Im}(f)) = \dim(\mathbb{R}^3) \rightarrow \text{Im}(f) = \mathbb{R}^3$   
 $f$  is SUPRAYECTIVE

$B = \begin{cases} \bar{e}_1 = (1, 0, 0)_B \\ \bar{e}_2 = (0, 1, 0)_B \\ \bar{e}_3 = (0, 0, 1)_B \end{cases}$

$\begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = -1 - 1 = -2 \neq 0$

$\underbrace{\dim(\text{Im}(f))}_3 + \dim(\text{Ker}(f)) = \underbrace{\dim(\mathbb{R}^3)}_3$

$\dim(\text{Ker}(f)) = 3 - 3 = 0 \rightarrow \text{Ker}(f) = \{0\}$   
 $f$  is INYECTIVE

$f: V_1 \rightarrow V_2$	$V_1 \neq V_2$	$V_1 = V_2$
Not Bijective	Homomorph.	Endomorph.
Bijective	Isomorph.	Automorph.

$f$  is INYECTIVE +  $f$  is SUPRAYECTIVE  $\rightarrow f$  is BIYECTIVE  $\rightarrow f$  is an AUTOMORPHISM

3.  $|F_B - \lambda I| = 0 \rightarrow \begin{vmatrix} 1-\lambda & 1 & 0 \\ 1 & -\lambda & 1 \\ 0 & 1 & 1-\lambda \end{vmatrix} = 0 \rightarrow (1-\lambda)[(\lambda^2 - \lambda) - 1] - (1-\lambda) = 0$

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$(\lambda - 1)(\lambda + 1)(\lambda - 2) = 0$



$$F_B = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$f(\bar{e}_1) \quad f(\bar{e}_2) \quad f(\bar{e}_3)$

$$\lambda_1 = -1 \rightarrow MO(-1) = 1$$

$$\lambda_2 = 1 \rightarrow MO(1) = 1$$

$$\lambda_3 = 2 \rightarrow MO(2) = 1$$

$$S(\lambda) = \text{Ker}(f - \lambda I)$$

$$f(\bar{e}_1) + f(\bar{e}_2) + f(\bar{e}_3) = (2, 2, 2)_B$$

$$f(\bar{e}_1 + \bar{e}_2 + \bar{e}_3) = (2, 2, 2)_B$$

$$f((1, 1, 1)) = (2, 2, 2) = 2(1, 1, 1)$$

$$1 \leq \dim(S(-1)) \leq MO(-1) \rightarrow \dim(S(-1)) = 1$$

$$1 \leq \dim(S(1)) \leq MO(1) \rightarrow \dim(S(1)) = 1$$

$$1 \leq \dim(S(2)) \leq MO(2) \rightarrow \dim(S(2)) = 1$$

$$S(-1) = \text{Ker}(f + I) \quad \begin{pmatrix} 2 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} x^1 \\ x^2 \\ x^3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$F_B + I \quad \bar{x} \quad \bar{0}$

$$\begin{aligned} 2x^1 + x^2 &= 0 \\ x^1 + x^2 + x^3 &= 0 \\ x^2 + 2x^3 &= 0 \end{aligned}$$

$$\begin{cases} x^1 = -\frac{1}{2}\alpha \\ x^2 = \alpha \\ x^3 = -\frac{1}{2}\alpha \end{cases} \quad \forall \alpha \in \mathbb{R}$$

$$S(-1) = \mathcal{L} \left\{ \underbrace{(1, -2, 1)}_{\bar{u}_1} \right\}$$

$$\begin{aligned} & \left( \begin{array}{ccc|c} 0 & 1 & 0 & x^1 \\ & & & 0 \end{array} \right) \quad x^2 = 0 \quad \begin{cases} x^1 = \beta \\ x^2 = 0 \end{cases} \quad S(1) = \mathcal{L} \left\{ \underbrace{(1, 0, -1)}_{\bar{u}_1} \right\} \end{aligned}$$

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$$S(2) = \text{Ker}(f - 2i) \quad \underbrace{\begin{pmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -1 \end{pmatrix}}_{F_B - 2I} \underbrace{\begin{pmatrix} x^1 \\ x^2 \\ x^3 \end{pmatrix}}_{\bar{x}} = \underbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}}_{\bar{0}}$$

$$\begin{cases} -x^1 + x^2 = 0 \\ x^1 - 2x^2 + x^3 = 0 \\ x^2 - x^3 = 0 \end{cases} \quad \begin{cases} x^1 = \gamma \\ x^2 = \gamma \\ x^3 = \gamma \end{cases} \quad \forall \gamma \in \mathbb{R}$$

$$S(2) = \mathcal{L} \left\{ \underbrace{(1, 1, 1)}_{\bar{u}_3} \right\}$$

4.

$$S(-1) = \mathcal{L} \left\{ \underbrace{(1, -2, 1)}_{\bar{u}_1} \right\}$$

$$S(1) = \mathcal{L} \left\{ \underbrace{(1, 0, -1)}_{\bar{u}_2} \right\}$$

$$S(2) = \mathcal{L} \left\{ \underbrace{(1, 1, 1)}_{\bar{u}_3} \right\}$$

Since  $\bar{u}_1, \bar{u}_2$  and  $\bar{u}_3$  are L.I.  
 Since  $\bar{u}_1, \bar{u}_2$  and  $\bar{u}_3$  are Eigenvectors

$$B' = \left\{ \underbrace{\bar{u}_1}_{ES(-1)}, \underbrace{\bar{u}_2}_{ES(1)}, \underbrace{\bar{u}_3}_{ES(2)} \right\}$$

Is a base of Eigenvectors of  $f$  in  $\mathbb{R}^3$

$$B' = \begin{cases} \bar{u}_1 = (1, -2, 1)_B = (1, 0, 0)_{B'} \\ \bar{u}_2 = (1, 0, -1)_B = (0, 1, 0)_{B'} \\ \bar{u}_3 = (1, 1, 1)_B = (0, 0, 1)_{B'} \end{cases}$$

$$B = \begin{cases} \bar{e}_1 = (1, 0, 0)_B = (1/6, 1/2, 1/3)_{B'} \\ \bar{e}_2 = (0, 1, 0)_B = (-1/3, 0, 1/3)_{B'} \\ \bar{e}_3 = (0, 0, 1)_B = (1/6, -1/2, 1/3)_{B'} \end{cases}$$

$$(1 \ 1 \ 1) \quad \text{Adj}(C)^t \quad C^{-1} = \begin{pmatrix} 1/6 & -1/3 & 1/6 \\ 1/3 & 0 & -1/3 \end{pmatrix}$$

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Is  $f$  diagonalizable? **Yes**

$f$  diagonalizable  $\iff \exists$  Base of Eigenvectors  $\iff \dim(S(\lambda_i)) = MO(\lambda_i) \quad \forall i$

$$B' = \left\{ \underbrace{\bar{u}_1}_{ES(-1)}, \underbrace{\bar{u}_2}_{ES(1)}, \underbrace{\bar{u}_3}_{ES(2)} \right\}$$

$$\begin{aligned} 1 \leq \dim(S(-1)) \leq MO(-1) &\longrightarrow \dim(S(-1)) = 1 \\ 1 \leq \dim(S(1)) \leq MO(1) &\longrightarrow \dim(S(1)) = 1 \\ 1 \leq \dim(S(2)) \leq MO(2) &\longrightarrow \dim(S(2)) = 1 \end{aligned}$$

$$F_{B'} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$f(\bar{u}_1) \quad f(\bar{u}_2) \quad f(\bar{u}_3)$   
in  $B'$

$$\begin{aligned} f(\bar{u}_1) = -\bar{u}_1 &\longrightarrow \bar{u}_1 \in S(-1) & -\bar{u}_1 &= (-1, 0, 0)_{B'} \\ f(\bar{u}_2) = \bar{u}_2 &\longrightarrow \bar{u}_2 \in S(1) & \bar{u}_2 &= (0, 1, 0)_{B'} \\ f(\bar{u}_3) = 2\bar{u}_3 &\longrightarrow \bar{u}_3 \in S(2) & 2\bar{u}_3 &= (0, 0, 2)_{B'} \end{aligned}$$

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